The Theory of Conceptual Fields

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The Theory of Conceptual Fields

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Abstract
The theory of conceptual fields is a developmental theory. It has two aims: (1) to describe and analyse the progressive complexity, on a long- and medium-term basis, of the mathematical competences that students develop inside and outside school, and (2) to establish better connections between the operational form of knowledge, which consists in action in the physical and social world, and the predicative form of knowledge, which consists in the linguistic and symbolic expressions of this knowledge. As it deals with the progressive complexity of knowledge, the conceptual field framework is also useful to help teachers organize didactic situations and interventions, depending on both the epistemology of mathematics and a better understanding of the conceptualizing process of students.

Introduction

Science is reduction. But not all reductions are fruitful. It is more or less accepted today that Piaget provided a superb contribution to the psychology of development, when behaviourists had not been able to do so. Nevertheless he was slowed down in the analysis of the mathematical contents by his fascination for logic and his hope to be able to reduce to logical structures the progressive complexity gained by children: for instance, his analysis of the 'formal stage' led him to identify the group of INRC transformations as the characteristic that would account for the understanding of proportionality by children. By doing so he did not pay enough attention to the contents that are specific to mathematics, namely the properties of functions.

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Simple proportion is a function of one variable; and two kinds of properties are essential:

- the isomorphic properties of linear functions:

  $f(x + y) = f(x) + f(y)$ and $f(ax) = af(x)$

  the constant coefficient $f(x) = kx$

- double proportion is the case when a variable is proportional to two other independent variables. The properties of bilinear functions are then relevant.

These properties would have better described the different competences emerging over a few years: the recognition and analysis of products and quotients of dimensions as is the case for the relationships between length, area and volume, or between measures in physics [for further details, see Vergnaud, 1983]. Nevertheless, in the theory of conceptual fields, I borrow from Piaget other important aspects of his work: first the concept of scheme, to which I give a larger interpretation than his, the thesis that knowledge is adaptation (accommodation and assimilation), as well as the overall Piagetian conception that action and representation play the main part in development.

I will first stress the importance of activity, schemes and situations for psychology, and then present a definition of a conceptual field as a set of situations and concepts. The concept of scheme also requires some attention, because it plays a crucial role in the analysis of the operational form of knowledge, as distinct from the predicative form. Finally, I will try to discuss different and complementary aspects of the concept of representation.

The Concept of Scheme

The concept of scheme was not introduced by Piaget: several philosophers of the 19th century mentioned it after Kant had introduced it and it was also used by several psychologists during the early decades of the 20th century, especially Revault d'Allonnes [e.g. 1915, 1920] and Janet [e.g. 1928] in France. However, Piaget was the first to provide concrete and convincing examples of its significance with his descriptions of early development in infants and young children. His book *La naissance de l'intelligence chez l'enfant* ('The origins of intelligence in the child') [Piaget, 1968a] is not only the 'invention' of infant cognitive development as a new field of research, but also the demonstration that gestures and perceptual acts are the empirical basis for its analysis. Therefore, the sequential organization of activity for a certain situation is the primitive and prototypical reference for the concept of scheme.

From this point of departure, several questions arise:

1 Is it possible to theorize about reasoning with the concept of scheme, and specifically about mathematical reasoning?
2 What is the role of schemes in the functioning of representation? Why and how are they components of representation?
3 What is their relation to other components like concepts, linguistic entities and symbols?

Activity, Schemes and Situations

The theory of knowledge as an adaptation process is essential; but what is it that adapts itself, and to what? The most reasonable answer to date is that what adapts are the forms of organization of activity, the schemes, and they adapt to situations. Therefore, the pair scheme/situation is conceptually more interesting and more powerful than the pair response/stimulus, and it is also more viable to describe and analyse behaviour and representation using the pair scheme/situation than the pair subject/object.

If the first reference for schemes is what Piaget (and most psychologists at the beginning of the 20th century) called 'sensory-motor' activity, the first theoretical question to be raised is how the gestural and perceptual actions undertaken in the real world are or become internal resources. It is not sufficient to say that schemes lie in the neurons and the genes, because it is hopeless to try to describe the organization of one single scheme as an organized sequence of active neurons, or as a set of genes, due to the billions of elements involved.

Moreover, this biological description misses the critical point of relating the internal and the external parts of activity, an essential point to promote an integrated framework of psychology. The best fruitful idea I can find is the idea, both Piagetian and Vygotskian, of interiorization (or internalization) of activity. This idea is well developed in Piaget's [1968b] book *La formation du symbole chez l'enfant* and in the first chapter of Vygotsky's [1962] *Thought and Language*. The paradox is that, in his radical critique of the Piagetian 'egocentric characteristic' of children's language, Vygotsky develops the view that egocentrism is rather 'a step in the internalization process' of dialogues, and offers the same idea of 'interiorized imitation' that Piaget understands as one of the first processes of representation.

How does this theory relate to the development of mathematical knowledge? Do we have examples of schemes in mathematics?

The very first example I will give is the scheme of counting objects. When children are able to count a small set of objects, they use three different repertoires of gestures: movements of arms and fingers, eye movements and words. The efficacy of the scheme depends on the one-to-one correspondence between these three activities and the set of objects in the physical world. It also relies on the ability to conclude the episode by wording the cardinal of the set, which is more than the last element of the set: cardinals can be added whereas last elements cannot. The concept of number is characterized by the additive property of cardinals, a property that equivalence and order relationships do not have. The concept of cardinal is implicit in the child's activity: it is a concept-in-action.

Another early example of scheme in mathematics is the perceptual activity used to recognize a building or a figure as symmetrical. The checking of symmetry can be more sophisticated than what 10-year-olds are able to do (for instance they would not check the equality of angles, or even the equality of distances to the axis of symmetry). But even when the control is loose, some invariant properties of symmetry are considered: they are also concepts-in-action.

What Is a Conceptual Field?

It is at the same time a set of situations and a set of concepts tied together. By this, I mean that a concept's meaning does not come from one situation only but from a variety of situations and that, reciprocally, a situation cannot be analysed with one concept alone, but rather with several concepts, forming systems.

As schemes and situations are the roots of cognitive development, and because concepts-in-action are essential parts of schemes (see definition below), the development of a conceptual field requires children's meeting and being faced with contrasting situations. Researchers also need to carefully analyse the different ways by which children tackle them. In this paper, I will give only one example, the conceptual field of additive structures. However, there are other good examples, like multiplicative structures, geometry of figures, positions and transformations, and elementary algebra.

There are two prototypical situations for addition: the binary combination of two parts into a whole ('4 boys and 5 girls for Kath's birthday party; how many children altogether?'), and the increase of an initial state ('Richard had 4 marbles, he wins 5; how many marbles does he have now?'), which can be better modelled by a unary operation, a function from the set of possible initial states into the set of final states.

The distinction between these two prototypes is made clear when one considers the variety of problems that can be generated.

In the first case, binary combination of two parts into a whole, only two classes of problems can be generated: knowing the two parts find the whole, and knowing the whole and one part find the other part.

In the second case, six classes of problems can be created: knowing the initial state and the transformation find the final state (by increasing or decreasing the initial amount), knowing the initial and the final states find the transformation, when the final state is bigger or smaller than the initial one, knowing the final state and the transformation find the initial state by increasing or decreasing the final state. Among these six classes of situations, four require a subtraction and only two an addition. Addition and subtraction are not merely inverse of each other.

There are large and significant differences in the success and failure met by children when they have to deal with the different classes of problems that can be generated starting from these two prototypes and from other cases like the quantified comparison of quantities ('Who has more, and how much? Find the compared quantity, or the referred one'), or else the combination and decomposition of transformations.

The simplest addition and subtraction situations can be dealt with by some 4-year-olds, and yet some situations requiring just one addition are still failed by the majority of 13- or 14-year-olds: 'Robert played two games of marbles; he remembers that he lost 7 marbles in the second game, but he does not remember what happened in the first game; by counting his marbles in the end, he finds that altogether, he won 5 marbles; what happened in the first game?'

Not only are there contrasts between situations, but also between schemes, i.e., between ways of tackling the situations. There are of course wrong ways, but one can also observe different valuable schemes for the same class of situations, depending for instance on the numerical values of the variables. Let us take the following situation: 'John has just won 7 marbles in playing with Meredith; now he has 11 marbles;

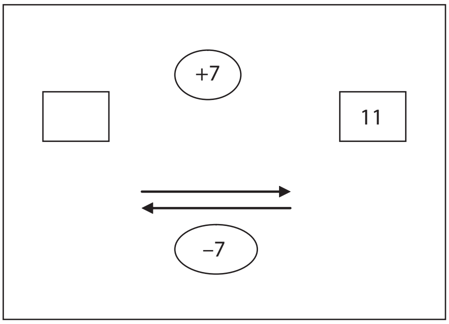

Fig. 1. Arrow diagram.

how many marbles did he have before playing?' Some children may of course subtract 7 from 11, others may count backwards from 11 down to 7 and count the number of digits, others may count from 7 upwards to 11, and still others may even make a hypothesis about the initial state (5 for instance), apply the increase of 7 marbles, find 12, which is too big, and correct their hypothesis. This last scheme is due mainly to the conceptual difficulty of inverting the increase of 7, by applying a subtraction of 7 to the final state. This operation of thinking requires a theorem-in-action:

If $T(I) = F$ then $I = T^{-1}(F)$

where I stands for initial state, F for final state, T for direct transformation, and T^{-1} for inverse transformation. This theorem can also be represented by an arrow diagram (fig. 1).

These two symbolic representations (the algebraic and the diagram) show that there are also contrasts between ways of symbolizing objects and relationships. Undoubtedly the algebraic representation would not be useful to children at the primary level, whereas the arrow diagram may at least convey the meaning of going forward and backward. This is essential, but does not solve the problem of understanding that +7 and –7 are inverse of each other. Children need several examples of the inverse character of the plus operation and the minus one. Several kinds of awareness are needed:

- You lose what you have just won; or you win what you have just lost.
- You go backwards as many steps as you have gone forwards and reciprocally.
- You give back to somebody the amount that he has lent to you or recover from somebody the amount of money you had lent him. A debt is therefore the inverse of a credit.

Not only are increase and decrease, or movements forwards and backwards empirical roots for positive and negative numbers, but also relationships between two persons (lending and borrowing) are examples of positive and negative numbers. This is important for the teaching of algebra and accountancy.

A provisional conclusion is that the development of a conceptual field involves situations, schemes and symbolic tools of representation. A comprehensive definition of representation is needed, but I will come to it only in the conclusion of this paper.

The Operational Form of Knowledge

Researchers who address the development of mathematical competencies cannot be satisfied with the view that mathematical words and sentences, as they appear in the textbooks or in the teachers' comments and explanations, would be a sufficient criterion to evaluate students' competencies. The test of their activity in situations is essential, particularly in novel situations, when they have to adapt their cognitive resources and face a problem never met before. The function of schemes, in the present theory, is both to describe ordinary ways of doing, for situations already mastered, and give hints on how to tackle new situations. Schemes are adaptable resources: they assimilate new situations by accommodating to them. Therefore the definition of schemes must contain ready-made rules, tricks and procedures that have been shaped by already mastered situations; but these components should also offer the possibility to adapt to new situations. On the one hand, a scheme is *the invariant organization of activity for a certain class of situations;* on the other hand, its analytic definition must contain open concepts and possibilities of inference. From these considerations, it becomes clear that schemes comprise several aspects defined as follows:

- The intentional aspect of schemes involves a goal or several goals that can be developed in subgoals and anticipations.
- The generative aspect of schemes involves rules to generate activity, namely the sequences of actions, information gathering, and controls.
- The epistemic aspect of schemes involves operational invariants, namely concepts-in-action and theorems-in-action. Their main function is to pick up and select the relevant information and infer from it goals and rules.
- The computational aspect involves possibilities of inference. They are essential to understand that thinking is made up of an intense activity of computation, even in apparently simple situations; even more in new situations. We need to generate goals, subgoals and rules, also properties and relationships that are not observable.

The main points I needed to stress in this definition are the generative property of schemes, and the fact that they contain conceptual components, without which they would be unable to adapt activity to the variety of cases a subject usually meets. I also feel the need to add several comments in what follows.

The dialectical relationship between situations and schemes is so intricate that one sometimes uses an expression concerning situations to refer to a scheme, for instance *high jumping,* or *solving equations with two unknowns,* as well as an expression concerning schemes to refer to a situation, for instance *rule of three situations* (the rule of three is a scheme, not a situation).

Another clarification concerns the relationship between concepts and theorems: the tie is so intricate that many researchers tend to confuse them. The difference is that a theorem can be true or false, because it is a sentence (or a proposition). A concept is not a sentence and therefore cannot be true or false, only relevant or not relevant. Another important point is that one may think a sentence is true that in fact is false; it is still a theorem-in-action. There is little difference, from the point of view of activity, between a true proposition and a false one considered as true.

The relationship between theorems and concepts is dialectical, in the sense that there is no theorem without concepts, and no concept without theorems. Yet the distinction is important in the theory of conceptual fields, because it is a theory of development: for instance, the analysis of additive structures shows that the concepts of addition and subtraction develop over a long period of time, through situations calling for theorems of very different levels.

The following example, in the domain of multiplicative structures, makes the difference very clear: 'suppose a student needs to find the quantity of flour that can be made with the corn production of a big farm: 182 tons. He has the information that one needs 1.2 kg of corn to make 1 kg of flour.' The scheme that comes to his mind after some time (which means that it is not a straightforward idea) consists in trying to find the ratio between 182 tons and 1.2 kg. This ratio is a scalar, a number that does not refer to a dimension, as it is the quotient of two magnitudes of the same kind (quantities of corn). But the choice to compute that ratio comes from the idea that it can be used to find the corresponding quantity of flour: it is the same ratio between the two quantities of corn (182 tons and 1.2 kg) and the corresponding two quantities of flour. Therefore, when one knows the ratio, the only thing to do is multiply it by the quantity of flour corresponding to 1.2 kg of corn:

$$F\ (\text{ratio} \times 1.2\ \text{kg}) = \text{ratio} \times F\ (1.2\ \text{kg})$$

This theorem is totally implicit and the process requires, also implicitly, that F (182 tons) be identified with F (ratio × 1.2 kg), and that F (1.2 kg) be identified with 1 kg of flour.

The calculation also requires the change of units, from tons to kilograms. The problem would be a little simpler if the production of the farm were given in kilograms, but it is not usual to do so for big productions.

The scalar ratio between 182 tons and 1.2 kg is a concept-in-action, not a theorem-in-action, but its use is invoked by the theorem.

The above scheme is not an algorithm, but it could be formalized into the following algorithm: 'in a four-term proportion, find the ratio between the two magnitudes referring to the same kind of quantity, and then apply it to the other quantity.' It is one of the practical burdens of mathematicians to discover or invent algorithms, and it is the job of students to learn them. Algorithms are schemes, but not all schemes are algorithms. The reason for this is that schemes do not have all the characteristics of algorithms: they lack 'effectivity,' namely the property to reach a solution, if there is one, in a finite number of steps. However, the organization of activity is very similar in schemes and algorithms. This similarity includes the fact that algorithms taught to students are often appropriated by them under a simplified organization; they can even change, after some time, into erroneous schemes.

The Operational Form and the Predicative Form of Knowledge

Complexity comes not only from doing, but also from putting something into words and saying it. Enunciation plays an essential part in the conceptualization process. One of the difficulties that students encounter when they learn mathematics is that some mathematical sentences and symbolic expressions are as complex as the situations and thought operations needed to deal with them. Some researchers even consider that the difficulty of mathematics is mainly a linguistic difficulty. This view is wrong, because mathematics is not a language, but knowledge. Still, understanding and wording mathematical sentences plays a significant role in the difficulties

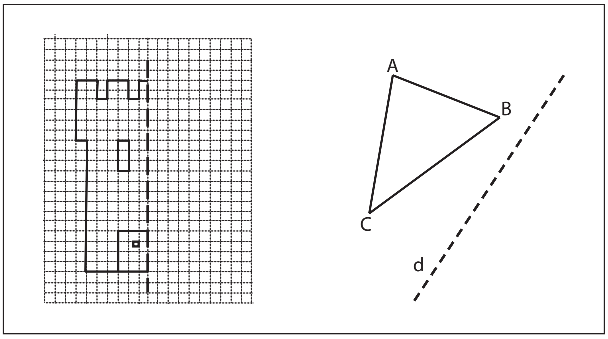

Fig. 2. Two situations for symmetry.

students encounter. To illustrate this point, let us take two situations in which students have to draw the symmetrical shape of a given figure. These situations contrast with each other, both from the point of view of the schemes that are necessary for the construction and from the point of view of the sentences that one may have to understand or produce on these occasions (fig. 2).

The first figure corresponds to a situation likely to be presented to 8- or 10-year-old students, in which they have to complete the drawing of the fortress symmetrically from the vertical axis. The second one could be typically given to 12- or 14-year-olds in France: construct a triangle symmetrical to triangle ABC in relation to d ('d' here refers to the dotted line).

In the first case, there are some coordination difficulties because the child needs to draw a straight line just above the dotted line, neither too high nor too low, and everybody knows that it is not that easy with a ruler; there is the same kind of awkwardness for the departure point and the arrival point. There are also conditional rules. For example, 'one square to the left on the part already drawn, one square on the right on the part to be drawn,' or else 'two squares down on the figure on the left, two squares down on the right,' or else 'one square to the right on the left figure, one square to the left on the one on the right,' starting from a reference point homologous to the point of departure on the left.

These rules are not very complex. Nevertheless they rely upon several concepts-in-action and theorems-in-action concerning symmetry and conservation of lengths and angles. As all angles are right angles and lengths are expressed as discrete units (squares), the difficulty is minimal.

In the second case, drawing the triangle A′B′C′, symmetrical to triangle ABC in relation to line d, is much more complex, with the instruments usual in the classroom (ruler, compass, set square). Even the reduction of the triangle to its vertices as sufficient elements to complete the task is an abstraction that some students do not accept easily because they see the figure as a non-decomposable whole. One step further, using d as the axis of symmetry for segments AA′, BB′, CC′, is far from trivial. Why draw a circle with its centre in A, and why should we be interested in the intersections of that circle with line d? One can also use a set square and draw a perpendicular line from A to d, measure the distance from A to d, go across line d to construct A′ at the same distance of A to d. But how can I think of the distance to be the same when there is no line yet?

The epistemological jump from the first to the second situation is obvious. But there are also big jumps between different sentences that are likely to be articulated on these occasions. I will use French rather than English because of the syntax of definite articles in French.

1 *La forteresse est symétrique* (‘The fortress is symmetrical’).
2 *Le triangle A′B′C′ est symétrique du triangle ABC par rapport à la droite d* (‘Triangle A′B′C′ is symmetrical to triangle ABC in relation to line d’).
3 *La symétrie conserve les longueurs et les angles* (‘Symmetry conserves lengths and angles’).
4 *La symétrie est une isométrie* (‘Symmetry is an isometry’).

Between sentence 1 and sentence 2, there is already a qualitative jump: the adjective *symétrique* moves from the status of a one-element predicate to the status of a three-element predicate (A is symmetrical to B in relation to C).

Between sentence 2 and sentence 3, the predicate *symétrique* is transformed into an object of thought, *la symétrie,* which has its own properties: it conserves lengths and angles. Nominalization (i.e., to form a noun from another word class or a group of words) is the most common linguistic process used to transform predicates into objects. In sentences 1 and 2, the idea of symmetry is a predicate (propositional function); in sentence 3, it has become an object (argument). Lower-case ‘s’ is the kind of symbol used by logicians for arguments, whereas upper-case ‘S’ is used for predicates. The two new predicates, Cl (conserving lengths) and Ca (conserving angles), are thus properties of this new object s.

$$\boxed{\begin{array}{l} S(f) \\ S(A'B'C', ABC, d) \end{array} \;\rightarrow\; Cl(s) \text{ and } Ca(s)}$$

When we move from sentence 3 to sentence 4, a new transformation takes place; the retention of lengths and angles then becomes an object of thought: isometry. This time the predicate is the inclusion relationship between two sets: the set of symmetries S and the set of isometries I:

$S \subset I$

The meaning of *la* in *la symétrie* in sentences 3 and 4 is the universal quantifier meaning. The *la* in *la forteresse* or *la droite d* in sentences 1 and 2 has a deictic value: ‘this fortress,’ ‘this line d.’ Obviously the correspondence between signified and signifiers is not a one-to-one correspondence on a lexical level.

Mathematical, scientific and technical texts, and more generally texts of a certain level (philosophy, literature, etc.) are full of such variations in the meaning of words, even though authors try to make them unambiguous.

Inevitably the succession of jumps in the operational and in the predicative forms of mathematical knowledge causes difficulties for students. Teachers are not aware enough of these jumps.

Representation

The different considerations and examples given above can be put together to theorize about the concept of representation. Behaviourists wanted to get rid of that concept, when they should have considered it as a central concept of psychology, like the concepts of force and movement in mechanics, or those of evolution and cell in life sciences. They thought it was impossible to have access to representation, but isn't this the actual situation of science? Newton did not have access to the attraction forces, neither Darwin to the succession of species, nor Mendel to genes.

Science is reduction, and the following ideas are a drastic reduction of psychological phenomena. But at least they offer possibilities to describe and analyse some important representational processes.

Four different components of representation can be distinguished, not as independent from one another, but as distinct in nature: (1) the flow of consciousness, (2) language and other sets of symbols, (3) concepts and categories and (4) sets and subsets of schemes.

The Flow of Consciousness

Every individual has some experience of the flow of consciousness. It is the most obvious proof of the existence of representation as psychological phenomena, even if it does not provide us with a fair and sufficient conception. This quasi-permanent flow of images (visual, auditory, kinaesthetic, somaesthetic) goes both with waking hours and dreaming, as well as with some consciousness of one's own gestures and words, sometimes only sketched in the mind. We cannot usually analyse this flow of percepts, ideas, images, words and gestures, but it testifies that representation works in a spontaneous and even irrepressible way. The flow of perception is an integral part of the flow of consciousness, also the flow of imagination, whether it is associated with perception or not. The fact that perception is a component of representation is important for psychological theory, as it is in the study of perception that one sees the essential role of concepts and categories in selecting information. The importance given here to consciousness is not contradictory with the existence of unconscious phenomena, or with the fact that there are privileged moments of sudden awareness, not reducible to the ordinary flow of consciousness.

Language and Symbols

Without words and symbols, representation and experience cannot be communicated. On top of that, thinking is often accompanied, or even driven, by linguistic and symbolic processes. Vygotsky had stressed that point very well. In the field of mathematics, numerical and algebraic notations play a very important part in conceptualizing and reasoning processes, although they are not concepts by themselves; musical notation is not music either, but symphonies would not be possible without it. What would mathematical thinking be without language and symbols? The predicative form of knowledge is obviously essential, even if it is not the first form of knowledge.

Concepts and Categories

Concepts and categories form the system with which we pick up information, with the aim of driving our activity in the most relevant way. This meaning of representation is not as direct as the first two, because it rests on the thesis that perception is an important component of representation, even when we have no words to be associated with the objects and relationships on which the organization of our activity relies. The word 'concept' is taken here in a wider sense than usual; it is normally restricted to explicit objects of thought, whereas here it is extended to concepts-in-action that are very often implicit in the course of activity. This is why I use the expression 'operational invariants' (instead of 'concept' and 'theorem') as much as I can. The distinction between conceptualizing and symbolizing is essential, up to the point where the understanding of words and sentences by different persons, particularly students and teachers, is not simply a binary relationship signifier/signified, but a ternary one with the interpretation privileged by operational invariants. A convincing example of such a process is the understanding of a formula like the formula for the volume of straight prisms:

$$V = A \times H$$

where V = volume of the prism, A = area of the base and H = height of the prism.

When they have to use it, students can read and interpret this formula in several ways. Here are some of their interpretations:

1 To calculate the volume I must know the area and the height and multiply one by the other.

2 To calculate the height I must know the volume and the area, and divide the volume by the area; this reading is already more difficult than the first one, as it is inverse.

3 Volume is proportional to the area when the height is held constant, and to the height when the area is held constant. This reading requires much more than understanding the operations of multiplication and division and the meaning of letters. It is not always mentioned in schoolbooks; yet it is the very reason for the formula.

Whatever the part of symbols may be in the conceptualizing process, one must not confuse concepts and symbols.

Systems of Schemes and Subschemes

Representation is a dynamic activity, not an epiphenomenon that would accompany activity without feeding it or driving it. Representation is neither a dictionary nor a library only, but also a functional resource: it organizes and regulates action and perception; at the same time, it is also the product of action and perception. Therefore, the operational form of knowledge must be considered as a component of representation. Schemes are essential: they organize gestures and action in the physical world, as well as interaction with others, conversation, and reasoning. Consciousness often accompanies activity, but only partially: it is especially related to goals and subgoals, assessing the relevance of the information grasped, controlling the effects of action. The structure of consciousness is different from the structure of activity: we are aware of the most relevant properties of objects, but more or less ignorant of the way activity is generated and the way subschemes are activated by superschemes. This hierarchical organization leaves room for improvisation and contingency: schemes and subschemes are often called upon by contingent aspects of situations; it is the reciprocal character of their adaptive function.

Conclusion

It is essential for psychologists to recognize the central function of activity in the development of representation, competences and concepts. Because language and symbols play an important role in the conceptualizing process, many researchers identify conceptualization and symbolization, as if the wording and symbolizing activity were sufficient roots of knowledge, particularly mathematical knowledge. This is not the case. The analysis of situations and schemes shows that the conceptualizing process already takes place in the simplest forms of activity (even without language): the reason is that no action can be efficient without the identification of some objects and their properties. Even more complex concepts, to gain sense and operationality, need to be contextualized and exemplified in situations. Therefore, from a developmental point of view, a concept is altogether: a set of situations, a set of operational invariants (contained in schemes), and a set of linguistic and symbolic representations.

There are specific characters in mathematical concepts that need to be treated as such. This is the main reason, both theoretical and practical, why it is more fruitful to use the framework of conceptual fields than logical structures to analyse the continuities and discontinuities of development in mathematics; also to imagine situations likely to push and help students to move along the multifaceted complexity of the field.

Finally, the operational form of knowledge and the predicative form are intertwined at all levels. There is no need to oppose one to the other; both are necessary to analyse the difficulties met by children and the way they can be overcome.

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